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THE EFFECT OF TEMPERATURE DEPENDENT PROPERTIES ON TRANSIENTS MEASUREMENT WITH INTRINSIC THERMOCOUPLE

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Abstract—Solution of the transient response of intrinsic thermocouples by Henning and Parker are expanded to include temperature dependent properties. Finite difference and Runge–Kutta numerical solutions are compared with the analytical solution. Results for thermocouples for which the temperature variability is significant are presented. The approach allows the calculation of large temperature steps.

An experimental method is described by which large step temperature increases can be simulated. The method also allows for large dimension substrate and thermowire, which reduce errors and slow-down real time performance to simplify measurements.

Experiment and calculations agree well after the very initial response. It is suggested that the experiment described is suitable for the measurement of the prompt response of intrinsic thermocouples.

NOMENCLATURE

a, dimensionless parameter, $= 1/(1+\beta/2\sqrt{\gamma})$;
C_p, specific heat;
G, correction factor in [1];
k, thermal conductivity;
R, radius of wire;
r, radial coordinate;
r^*, dimensionless radial coordinate, $= r/R$;
S, temperature in substrate;
S^*, dimensionless temperature in substrate, $= (S-T_\infty)/(S_0-T_\infty)$;
S_0, initial elevated temperature of substrate;
T, temperature in wire;
T^*, dimensionless temperature in wire, $= (T-T_\infty)/(S_0-T_\infty)$;
T_∞, ambient temperature;
t, time;
t^*, dimensionless time, $= t\alpha_s/R^2$;
x, axial coordinate in wire from substrate;
x^*, dimensionless axial coordinate in wire, $= x/R$.

Greek symbols

α, thermal diffusivity;
β, ratio of conductivities, $= k_w/k_s$;
γ, ratio of thermal diffusivities, $= \alpha_w/\alpha_s$;
θ, temperature of junction wire-substrate;
θ^* dimensionless temperature of junction, $= (\theta-T_\infty)/(S_0-T_\infty)$;
ρ, density;
ϕ, coefficient of linear variable conductivity.

INTRODUCTION

THE PROBLEM of intrinsic thermoelements has been dealt with intensively. Henning and Parker [1] derived and solved a theoretical model and performed experiments. More recently, Bickle and Keltner [2, 12] did an extensive study, reviewed and summarized previous work, as well as developed means for analyzing temperature–time responses of intrinsic (and other) thermoelements [3]. The advantages of intrinsic thermocouples for the measurement of temperature transients, particularly in the case of internal heating are overwhelming. The partially zero order response, i.e. instantaneous step output for a step input, make them indispensible for short time reactions.

For systems with potential fast transients of large amplitude such as nuclear reactors, the immediate information of the exact real temperature is vital. Crucial immediate decisions must be made based on temperature data. Intrinsic thermocouples are particularly suited to accomplish the measurement of rapid transient surface temperature change of conducting solids. Many mechanical systems subject to temperature transients are constructed of conducting solids—making the intrinsic thermocouple an essential temperature sensor.

The previously mentioned works use essentially the same basic model, as is done here. In the previous models the heat conduction is assumed to be temperature independent and constant. Parker and Henning limit their analysis to small temperature steps of about 50°F. Within such a temperature range and with an empirical correction factor they use ("G") and the modification of Giedt and Nunn [1] results are satisfying. Bickle and Keltner are not limiting the temperature span. They developed the "deconvolution" [2, 4] method which is essentially a time increment calibration method. Since it is a calibration method, it will compensate for all changing variables, but may require a somewhat elaborate calibration.

The present paper expands the Henning and Parker model to include temperature dependent properties. However, all of the emphasis is put on the variation of the thermal conductivity with temperature. The heat capacity per unit volume, ρC_p, varies little for metals [1]. The solution required a numerical approach. The

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numerical method allows the variation of properties other than the thermal conductivity as necessary. A measurement technique is presented which is not limited to electrical discharge, as in the previous work. The preheated element enables essentially unlimited temperature steps at almost perfect step functions. The hitherto limitations of size, due to limited energy storage and step discharge ability have been overcome. It is possible to determine the prompt response as well as the dynamic response with relative simple means.

THEORETICAL MODEL AND NUMERICAL METHOD

The theoretical model is discussed in [1, 2]. The governing equation for the substrate (fuel element) is:

$$\frac{\partial S^*}{\partial t^*} = \frac{\partial^2 S^*}{\partial r^{*2}} + \frac{2}{r^*}\frac{\partial S^*}{\partial r^*}. \tag{1}$$

In deviation from the previous derivations we consider only a single wire for which only axial conduction is permitted:

$$\frac{\partial T^*}{\partial t^*} = \gamma \frac{\partial^2 T^*}{\partial x^{*2}}. \tag{2}$$

In the case of two wires there will be a separate equation (2) for each wire. The net thermoeffect will be a resultant of the combined partial e.m.f.'s of each wire in contact with the substrate [5].

The boundary conditions assume a step temperature change in the substrate from S_∞ to S_0 at time 0.

$$S^*_{t<0} = T^*_{t<0} = T^*_{t,\infty} = 0 \tag{3}$$

$$S^*_{t=0} = 1. \tag{4}$$

The heat flux at the interface, assuming the half sphere under the thermowire has neither heat capacity nor resistance to energy transfer yields:

$$\left.\frac{\partial S^*}{\partial r^*}\right|_{t,1} = \frac{r}{2}\left.\frac{\partial T^*}{\partial x^*}\right|_{t,0} \tag{5}$$

and the wire temperature at great distance from the junction does not change:

$$T^*(t, \infty) = 0. \tag{6}$$

The analytical solutions assuming constant conductivity were presented by Parker and Henning [1].

To enable a variable thermal conductivity coefficient a numerical approach was used. Two solution methods were applied and compared. The thermal conductivity is assumed to vary linearly with temperature. The finite difference method utilized first central finite differences for the space derivations and first forward finite differences for the time derivative.

$$\frac{T_{i+1,n} - T_{i,n}}{\Delta t} = \frac{k_{0w} - \phi_w(T_{i,n} - S_\infty)}{\rho_w c_{pw}}\left[\frac{T_{i,n-1} - 2T_{i,n} + T_{i,n+1}}{(\Delta x)^2}\right] - \frac{\phi_w}{\rho_w c_{pw}}\left(\frac{T_{i,n+1} - T_{i,n-1}}{2\Delta x}\right)^2 \tag{7}$$

$$\frac{S_{i+1,n} - S_{i,n}}{\Delta t} = \frac{k_{0s} - \phi_s(S_{i,n} - S_\infty)}{\rho_s c_{ps}} \times \left[\frac{S_{i,n-1} - 2S_{i,n} + S_{i,n+1}}{(\Delta x)^2} + \frac{2}{m\Delta r + R}\left(\frac{S_{i,n+1} - S_{i,n-1}}{2\Delta r}\right)\right] - \frac{\phi_s}{\rho_s c_{ps}}\left(\frac{S_{i,n+1} - S_{i,n-1}}{2\Delta r}\right) \tag{8}$$

and for the heat flux at the interface:

$$k_s 2\pi\left(R + \frac{\Delta r}{2}\right)^2\left(\frac{S_{i,2} - S_{i,1}}{\Delta r}\right) - k_w \pi R^2\left(\frac{T_{i,0} - T_{i,1}}{\Delta x}\right) = \rho_s c_{ps}\tfrac{2}{3}\pi\left(R + \frac{\Delta r}{2}\right)^3\left(\frac{S_{i+1,1} - S_{i,1}}{\Delta t}\right) + R^2\pi\rho_w c_{pw}\frac{\Delta x}{2} \times \left(\frac{T_{i+1,0} - T_{i,0}}{\Delta t}\right)\tfrac{2}{3}k_s\pi R^2\left(\frac{S_{1,2} - S_{1,1}}{\Delta r}\right) = k_w\pi R^2\left(\frac{T_{1,0} - T_{1,1}}{\Delta x}\right). \tag{9}$$

The stability criterion applied was [6]

$$\alpha_w \Delta t/(\Delta x)^2 = \alpha_s \Delta t/(\Delta r)^2 = 0{\cdot}25. \tag{10}$$

The equations were programmed and numerical solutions, discussed later, obtained [7, 8].

The second method applied was the Runge–Kutta. First central finite differences for the space derivations were used to form a set of time dependent first order differential equations. Available subroutines in the IBM 360/65 were used for the numerical solutions. The stability criterion for the Runge–Kutta was chosen to be 0·25 rather than 0·7 which would have been sufficient [9]. High order terms were neglected.

The thermal conductivity was assumed to vary linearly between known points. It was amazing to discover a wide spread of properties data in the literature for the different materials. For varied properties data an arbitrary decision was made as to which data to use. Material properties are summarized in Table 1.

EVALUATION

Figures 1 and 2 show the results of the calculations for constant properties in comparison to the analytical solution. It is obvious from these figures that the finite difference results and the Runge–Kutta calculations with 1000 equations yield satisfactory results as compared with the analytical model except at the very beginning. All the later calculations were made with the finite difference and Runge–Kutta method with one thousand differential equations. It may be worthwhile noting that the initial numerical evaluations tend to yield lower temperatures than the analytical calculations. It has been shown by Henning and Parker that the uncorrected analytical results tend to be initially higher than experimental results [1].

The efforts were concentrated on the cases in which the change in the coefficient of conductivity are of significance. As expected the effect of the temperature dependent conductivity is more pronounced the more

Table 1.* Approximate physical property data, $k = k_0 - \phi(T \text{ or } S - T_\infty)$

	Temperature range		k_0		ϕ_0		ρ		C_p	
	(°F)	(°C)	$\left(\frac{\text{Btu}}{\text{h.ft.°F}}\right)$	(W.m^{-1}.K^{-1})	$\left(\frac{\text{Btu}}{\text{h.ft.°F}^2}\right)$	(W.m^{-1}.K^{-2})	$\left(\frac{\text{lb}}{\text{ft}^3}\right)$	(t.m^{-3})	$\left(\frac{\text{Btu}}{\text{lb.°F}}\right)$	(kJ.kg^{-1}.K^{-1})
Alumel			17·2	29·8			535	8·6	0·125	0·52
Aluminum	70–700	21–371	130	225	0·03	0·09	169	2·7	0·214	0·90
Chromel P			11·1	19·2			545	8·7	0·107	0·45
Constantan	212†–950	100†–500	13	22·5	−0·019	−0·059	555	8·9	0·094	0·39
Copper	70–700	21–371	229	39·6	0·032	0·1	559	8·9	0·092	0·39
Inconel X	27–1070	0–550	7·6	13·2	−0·0068	−0·021	525	8·4	0·106	0·44
Platinum	70–800	21–427	41·0	71	0·0014	0·0044	1340	21·4	0·0316	0·13
Steel 1% C	70–700	21–371	26·3	45·5	0·002	0·006	487	7·8	0·113	0·47
(Multiplier)				1·731		3·116		0·016		4·187

*Values from sources 1, 10, 11 and *estimated* values *as used* in the program. (Values should *not* be considered for other usage.)
†Used from room temperature.

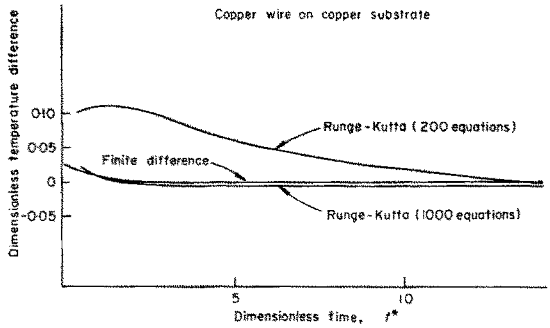

FIG. 1. Dimensionless temperature difference (analytical results minus finite difference and Runge–Kutta).

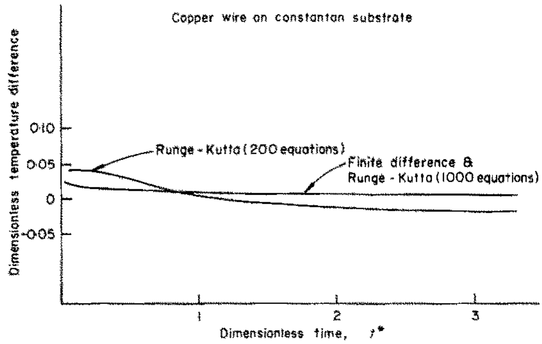

FIG. 2. Dimensionless temperature difference (analytical results minus finite difference and Runge–Kutta).

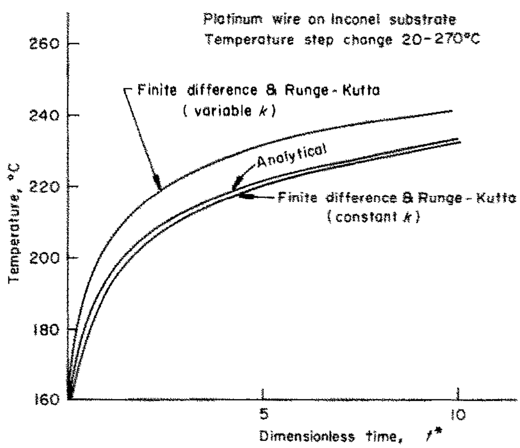

FIG. 3. Temperature (analytical, finite difference and Runge–Kutta results).

the coefficient of conductivity is temperature dependent. The effect diminishes for small temperature steps, as has been shown previously [1].

The largest effect was found in the combination of a Platinum wire on an Inconel substrate. For a 250°C temperature jump the difference between constant and variable conductivity is up to 10°C or about 5 per cent (Figs. 3 and 4). Significant differences are accounted for in cases of copper wire on a constantan substrate (Fig. 5). These are material combinations which are likely to be used and are therefore of applied interest. In other cases such as copper on steel the effect of variable conductivity diminishes in the error of the calculations.

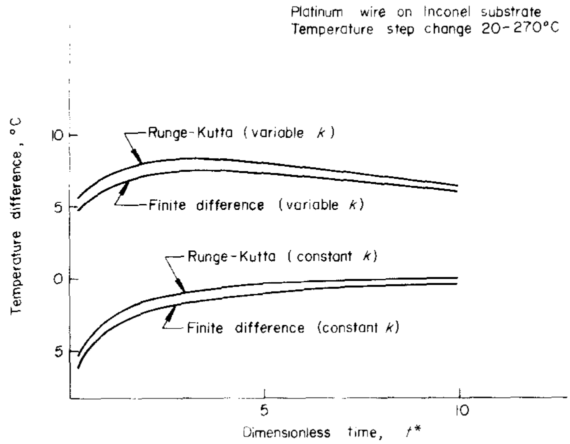

FIG. 4. Temperature difference (analytical results minus finite difference and Runge–Kuttà).

touches simultaneously. Between successive experiments the wire was allowed to cool down to room temperature. The temperature of the substrate at the touching point was checked with a bead iron–constantan thermocouple and found in good agreement with the results of the copper–constantan couple.

Each experiment was repeated several times, there were only minor differences between the different runs. Occasionally an experiment registered lower temperatures than several others. This was attributed to poor thermal contact. Experiments were repeated until several results converged at the high temperature, while only a few were scattered at lower temperatures.

The described arrangement and procedure simulate well a step heating of the substrate. The substrate is preheated to a predetermined temperature. When touched with the cold wire the heating of the junction

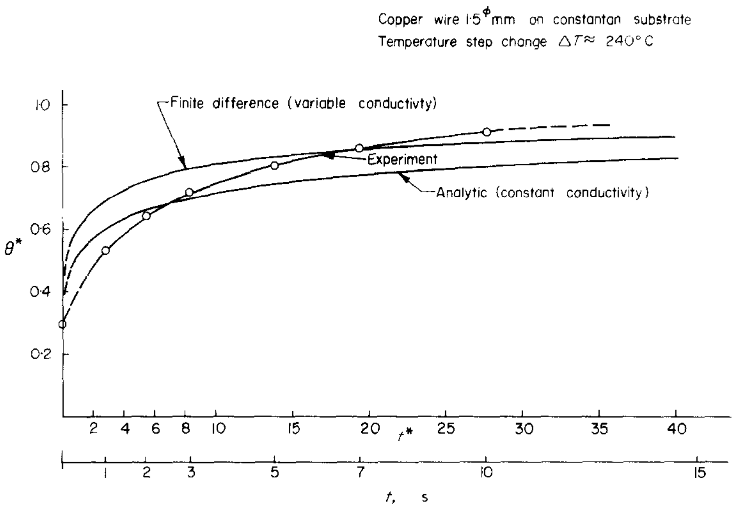

FIG. 5. Dimensionless thermocouple junction temperature.

EXPERIMENT

Experiments were performed with substrates made of 14 mm dia chromel, 10 mm alumel and 6 mm constantan (see Acknowledgement). The experiment utilizing the constantan bar is described in detail. A cylinder of about 6 cm length was flattened on one side to create a flat face on one side of 4 mm width along the cylinder. The flat face was polished. At one end of the cylinder a constantan wire was fitted into a central hole and hammered in. The other end of the cylinder was put into the cavity of a soldering iron to be heated.

A 1·5-mm dia copper-wire was flattened and polished at the end. The other copper-wire end, and the constantan wire, were connected to a plotter. The plotters abscissa had a time base of 1–5 cm/s. A schematic drawing of the experimental set-up is depicted in Fig. 6.

The copper wire was hand held and touched to the flattened side of the constantan substrate at a predetermined position. Special care was taken to touch the wire perpendicularly so that the full cross-section and the measurement begin simultaneously. The relative large dimensions of substrate and wire provide several advantages. The most prominent advantage is the slow-down of the transient in real time. The large diameter wire reduced secondary effects such as lateral heat losses. The substrate as a whole is essentially at steady state. Electrical (heating) and other transients are avoided. All these advantages provide for an accurate undisturbed measurement, at almost any temperature step.

Results for copper wire and constantan substrate along with the results of the calculations are shown in Fig. 5. The dimensionless temperature was calculated by assuming the temperature reached steady state at about 20 s, which corresponds to a dimensionless time of 55. This time period is not sufficient to achieve real equilibrium corresponding to the model assumptions. However, at such a time a steady state is reached which is due to lateral losses from the wire and the effect of the final dimensions of the substrate. Thus the results for the experiment are shown too high to agree with the theoretical results. It has been shown previously [1]

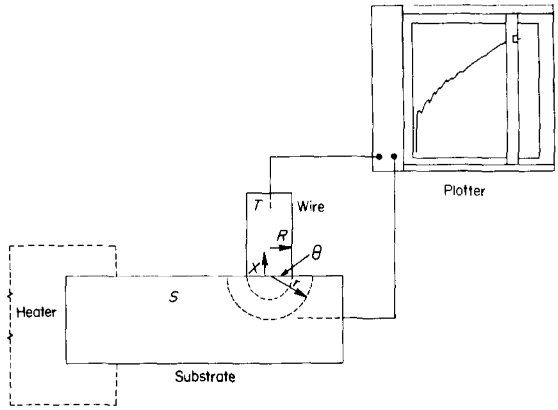

FIG. 6. Schematic of the experiment.

that the initial experimental results tend to be lower than the calculated results. The theoretical "a" value for copper constantan is 0·32. The extrapolated prompt response is about 0·30 with an estimated error of 0·01.

CONCLUSIONS

The numerical approach to the solution of the transient response of intrinsic thermocouples is satisfactory. Both the finite difference and the Runge–Kutta methods compare well with the analytical solution. The numerical approach enables the inclusion of properties variability with temperature. The numerical approach enables the calculation of large temperature steps in which the change of the material properties is significant. For some material combination the difference in the calculated actual temperature is of the order of several percent.

The Runge–Kutta method with sufficient number of equations—here 1000—is superior to the finite difference method for a well defined geometry and semi-infinite substrate. The finite difference method enables to account for less defined geometries, as may occur in actual cases. Finite difference method also allows to improve the model with respect to the half sphere under the wire [12].

The experiment supports the theoretical model. Except at the very beginning, where all uncorrected calculations predict a higher temperature than actual, results compare well with the numerical calculations. Deviations in the approach to steady state temperature are attributed to secondary losses which the model does not account for.

The initial disagreement of experiment and calculation may be attributed to the fact that the model neglects the half sphere under the wire. The model thus simulates a larger area through which heat is transferred than actually available. The model also does not account for the heat required to sustain the temperature of the neglected half sphere. Both effects contribute to a higher calculated temperature than the measured temperature. The high experimental temperature shown in Fig. 5, as compared to the finite difference calculated temperature, is due to the normalization of the temperature scale assuming the terminal temperature ($\theta = S_0$) has been reached in the finite time of the experiment at no losses to the environment.

The described experimental method is useful in simulating large temperature steps. The method enables to use large dimension substrate and wire, which in turn slow down the actual time response thus simplifying the experimental set-up and improving accuracy.

With an improved attachment method, e.g. mechanical rather than hand held, and improved environmental control, the prompt response of intrinsic thermoelements could be measured accurately. Lateral insulation of substrate and wire will better simulate the model throughout the range. The insulation importance increases the larger the temperature step.

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L'INFLUENCE DES PROPRIETES DEPENDANT DE LA TEMPERATURE SUR LES MESURES EN TRANSITOIRE A L'AIDE DE THERMOCOUPLES INTRINSEQUES

Résumé—La solution de Henning et Parker de la réponse transitoire des thermocouples intrinsèques est étendue au cas de propriétés dépendant de la température. Les solutions numériques de différences finies et de Kutta–Runge sont comparées à la solution analytique.

Des résultats sont présentés pour des thermocouples dont les variations de température sont importantes. La méthode permet de calculer de grands écarts de température.

Une méthode expérimentale est décrite qui permet de simuler des échelons importants de température. La méthode permet aussi de grandes dimensions de support et de fil, qui réduisent les erreurs et ralentissent le temps réel de fonctionnement ce qui simplifie les opérations de mesure.

Les expériences et les calculs se trouvent en bon accord après la réponse initiale elle-même. Il est suggéré que l'expérience décrite est adaptée à la mesure de la réponse rapide des thermocouples intrinsèques.

EINFLUSS TEMPERATURABHÄNGIGER EIGENSCHAFTEN BEI INSTATIONÄREN MESSUNGEN MIT EIGENLEITENDEN THERMOELEMENTEN

Zusammensfasung—Die Deutung instationärer Meßergebnisse von eigenleitenden Thermoelementen durch Henning und Parker wird auf temperaturabhängige Eigenschaften ausgedehnt. Lösungen mit kleinen Differenzen und numerische Runge-Kutta-Lösungen werden mit analytischen Lösungen verglichen. Ergebnisse von Thermoelementen, für die die Temperaturveränderlichkeit kennzeichnend ist, werden aufgeführt. Das Vorgehen erlaubt die Berechnung von großen Temperaturintervallen. Eine experimentelle Methode wird beschrieben, bei der große Schritte der Temperaturzunahme simuliert werden können. Die Methode gestattet große Abmessungen bei Substraten und Thermodrähten, die die Fehlermöglichkeiten reduzieren und das Realzeitverhalten verlangsamen, was die Messungen vereinfacht. Versuch und Rechnung stimmen nach den allerersten Meßwerten sehr gut überein. Es wird darauf hingewiesen, daß der beschriebene Versuch sich zur Messung des unmittelbaren Ausgangssignals von eigenleitenden Thermoelementen eignet.

ВЛИЯНИЕ СВОЙСТВ, ЗАВИСЯЩИХ ОТ ТЕМПЕРАТУРЫ, НА КРАТКОВРЕМЕННОЕ ИЗМЕРЕНИЕ С ПОМОЩЬЮ ТЕРМОПАРЫ, ВВЕДЕННОЙ ВНУТРЬ ТЕЛА

Аннотация — Проблема изучения чувствительности термопар, введенных внутрь тела, Хеннином и Перкером дополнена изучением свойств, зависящих от температуры.

Решения, полученные методом конечных разностей и численным методом Рунге-Кутта, сравнивались с аналитическим решением.

Представлены результаты измерения значительных температурных колебаний при помощи термопар.

С помощью аппроксимации возможен расчет больших скачков температуры. Описан экспериментальный метод, с помощью которого можно моделировать получение больших скачков температуры.

Для упрощения измерений в методе рекомендуется использовать подложку и проволоку больших размеров, что уменьшает ошибки и замедляет реальный процесс измерения.

Эксперименты и расчет сразу показали хорошее согласие.

Предполагается, что с помощью описанного эксперимента можно измерять мгновенную реакцию термопар, введенных внутрь тела.